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Production of Semi Quark Gluon Monopole Plasma by Glasma Decay

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Using the standard Lagrangian of gluons and a model of dual superconductor for magnetic monopoles, we calculate the number densities of the gluons and monopoles produced by the decay of background color electric E and magnetic B fields (glasma). We find that gluons are dominant decay products when the initial values of the gauge fields are large such that $gE = gB > (0.5\text{GeV})^2$, while they are suppressed and monopoles are dominant decay products when the initial values are small such that $gE = gB < (0.4\text{GeV})^2$. The feature of the gluon dominance at large $gE = gB$ and the monopole dominance at small $gE = gB$ is similar to the one of thermalized quark gluon monopole plasmas proposed recently, if we identify $\sqrt{gE} = \sqrt{gB}$ as temperatures of the plasmas. The identification is suggested by the fact that the energy densities of the gluons and monopoles are proportional to the initial values $(gB)^2 = (gE)^2$, while the energy densities of the plasmas are proportional to T^4 . The feature of the gluon dominance in the glasma with large saturation momenta has been derived in classical statistical field theories, while the feature of the monopole dominance has not yet derived. Although the model of the monopoles is phenomenological, our analysis suggests that the monopoles play important roles in the decay of the glasma with small saturation momenta, to which classical statistical field theories are not applicable.

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I. INTRODUCTION

Quark gluon plasmas (QGPs) have been produced by high energy heavy ion collisions. They have been extensively explored and shown to be thermalized[1] in a very short period $< 1\text{fm}/c$ after the collisions. The plasmas are expected to approach to the ideal gas at high temperatures $T \gg 1\text{GeV}$. In such a temperature the plasmas are composed of weakly coupled quarks and gluons. On the other hand, the plasmas are composed of strongly coupled quarks and gluons[2] at low temperatures, e.g. $T < 0.7\text{GeV}$. With further decrease of the temperatures, a phase transition takes place at a critical temperature $T_c \sim 0.16\text{GeV}$ and the quarks and gluons are confined in hadrons.

At high temperatures, the weakly coupled quarks and gluons are quasi-particles in the plasmas. On the other hand, it is not clearly understood what are quasi-particles in the plasmas of the strongly coupled quarks and gluons at low temperatures, for example, at the temperature $T = (1 \sim 2)T_c$ where coupling strength is given by $\alpha_s = g^2/4\pi \sim 1$ with gauge coupling constant g . But, the number f_{eff} of quasi-particles defined such that $f_{\text{eff}} \equiv 30\epsilon/(\pi^2 T^4)$ with energy density ϵ has been shown in the lattice gauge theories[3] to be rapidly suppressed in such strong coupled plasmas as the temperatures approach to the critical temperature T_c .

It has recently been proposed[4] that quasi-particles of the strong coupled QCD plasmas are magnetic monopoles[5] in addition to quarks and gluons. According to the model [4], quarks and gluons are dominant components in the plasmas at high temperature $T > 3T_c$, while the monopoles are dominant components at low temperature $T = (1 \sim 2)T_c$. In other words, the effective dynamical degrees of freedom of quarks and gluons are suppressed in the low temperature $T \simeq (1 \sim 2)T_c$, and instead the monopoles become dominant. It was pointed out[6] before the proposal[4] that the monopoles play important roles in strongly coupled QGPs with temperatures near T_c . For example, they play a role of making small shear viscosity of the plasmas[6]. Furthermore, the monopoles play the role of quark confinement[7, 8] at the temperatures $T < T_c$. The dominance of the monopoles at low temperature $T \simeq (1 \sim 2)T_c$ and the dominance of the quarks and gluons at high temperature $T > 3T_c$ is a characteristic feature of the model of the thermalized quark gluon monopole plasmas (QGMPs).

In this paper we discuss prethermalized states of monopoles and gluons. They are the states produced by the decay of homogeneous color electric E and magnetic B fields. The presence of such classical gauge fields (glasma) produced by the high energy heavy ion collisions has been discussed in a model of color glass condensate[9]. Although these monopoles and gluons interact with each other and would be thermalized after their production, the states we discuss in the paper are prethermalized states of the non-interacting gluons and monopoles. We do not address how the states are thermalized, but address which ones are dominant decay products of the gauge fields, gluons or monopoles.

The field strengths of the glasma depend on saturation momentum Q_s of the color glass condensate. The strong gauge fields with large Q_s have small gauge couplings $\alpha_s \ll 1$, while the weak gauge fields with small Q_s , but still larger than Λ_{QCD} have large gauge couplings $\alpha_s \sim 1$. There are some reliable methods with which the decays of the strong gauge fields can be analyzed. They are classical statistical field theories[10–12], Schwinger mechanism[13, 14] or classical numerical simulations[15–18] in gauge theories of quarks and gluons. But there are no reliable methods with which the decays of the weak gauge fields can be analyzed since α_s is large. We need to use some non-perturbative methods or models of strongly coupled quarks and gluons. Here we use a model of dual superconductors[19, 20] in QCD as an effective field theoretical model of the monopoles; they are assumed to be quasi-particles of strongly coupled gluons.

The phenomenological model of the monopoles has been used for the analysis of quark confinement in QCD vacuum. It is natural to apply it to the analysis of the glasma decay which leads to strongly coupled QGPs, in which perturbative analysis is not valid. The non-perturbative analysis based on the phenomenological model would be valid for example in a range $1 > \alpha_s > 0.5$ (or $T \simeq (1 \sim 2)T_c$) where the glasma still hold the coherence as classical fields and the monopoles do not strongly couple with each others. We note that the gluon occupation number of the glasma is of the order of α_s^{-1} and the magnetic couplings of the monopoles are also given by α_s^{-1} . For the analysis of the glasma decays, we use both of the model and the gauge theories of gluons in the whole range of the gauge couplings $1 > \alpha_s > 0$. It turns out that the glasma mainly decay into monopoles when the gauge couplings are strong such as $1 > \alpha_s > 0.5$, while they mainly decay into gluons when they are weak such as $0.3 > \alpha_s > 0$.

The prethermalized states of the decay products have a similar feature to that in the QGMPs mentioned above. Namely, the gluons are dominant decay products of the strong gauge fields, while the monopoles are dominant ones of the weak gauge fields. Thus, the temperatures of the thermalized plasmas produced by the strong gauge fields is high, while the temperatures of the thermalized plasmas produced by the weak gauge fields is low. Thus, if we identify the initial values of \sqrt{gB} or \sqrt{gE} as a temperature, the dominant components of the prethermalized plasmas are similar to the ones in the QGMPs. The identification \sqrt{gB} or $\sqrt{gE} \sim T$ is suggested by the fact that the energy densities of the prethermalized gluons and monopoles are given by the initial values $(E^2 + B^2)/2$ of the gauge fields, while those of the QGMPs are roughly given by T^4 . There is a duality such that the gluons play dominant roles in the glasma with large Q_s or QGP plasmas with high temperatures, while the monopoles do dominant roles in the glasma with small Q_s (but still larger than Λ_{QCD}) or QGP plasmas with low temperatures (but still larger than T_c).

We assume in the present paper that the color glass condensates are still present even when Q_s is small just as $Q_s = (1 \sim 2)\Lambda_{QCD}$ and that they are described by classical color gauge fields E and B since the gluon occupation number is given by $\alpha_s(Q_s)^{-1} \simeq 1 \sim 2$. We call such glasma strongly coupled glasma, while we call the glasma with large Q_s weakly coupled glasma. In the next section, we describe how the monopoles play roles in the decay of the strongly coupled glasma. In the section III, we describe the applicability of our model by the use of which the productions of gluons and monopoles are discussed. Actual models of gluons and monopoles are presented in section IV. The evolution equations of the number densities of gluons and monopoles are presented in section V. Our results are shown in the section VI. In the final section we present our discussions and conclusion.

II. MAGNETIC MONOPOLES

We explain the role of the color magnetic monopoles in the decay of the glasma. Their decay has been mainly discussed using the classical statistical field theory[10–12], Schwinger mechanism[13, 14] or classical numerical simulations[15–18]. Although the classical statistical field theory is well controlled method, it is only applicable to the very weakly coupled glasma, that is, glasma with sufficiently large saturation momenta Q_s for the gauge coupling to be extremely small $g(Q_s) \ll 1$. However, the theories are not applicable for the glasma with realistic small gauge couplings $g(Q_s) \sim O(1)$ [21]. On the other hand, the Schwinger mechanism is applicable even for the moderately strongly coupled glasma. The pair creation of gluons arise according to the mechanism, which makes the color electric fields decrease. But the magnetic fields of the glasma hardly decay in the mechanism. This is because the pair creations of gluons do not make the magnetic fields decrease. Similarly, the numerical simulations using classical equations of motion are only applicable for sufficiently strong gauge fields for gluons to keep the coherence. But with the expansion of the glasma, the coherence is not kept since the gluons become dilute. Furthermore, the classical treatments including the classical statistical studies does not make clear what are quasi particles after the decay of the gauge fields. In this way, it is not efficient to apply these methods to the analysis of the decay of the strongly coupled glasma with saturation momenta such as $\alpha_g(Q_s) = 1/2 \sim 1$.

Obviously, the magnetic monopoles make the magnetic fields efficiently decay in monopole plasmas. They also play the role of confining quarks and gluons[7, 8] in the strongly coupled QCD vacuum. The monopoles are well defined objects in QCD when the gauge couplings are large, since the magnetic charge is so small that their mutual interactions are small. It is expected that the monopoles play important roles in the glasma with small Q_s or QGPs

with temperatures near T_c . Actually, it was pointed out[6] that the monopoles play important roles in the strongly coupled QGPs with the low temperatures as well as in QCD vacuum. In particular, it has recently been discussed[4] in the realistic analysis of high energy heavy ion collisions that they are present even at $T > T_c$ and play significant roles in the QGMPs. In the discussions they are treated simply as point particles with magnetic charges g_m satisfying the Dirac quantization condition $g_m = 4\pi/g$. But, their production mechanism in heavy ion collisions and their properties (masses or spins) in the thermalized states are still not well-known. Thus, it is reasonable to apply a phenomenological model of the magnetic monopoles to the analysis of the decay of the strongly coupled glasma with small Q_s . It is the model of the dual superconductors. It has been extensively discussed to analyze strongly coupled QCD vacuum. The model is phenomenological and our production mechanism of the monopoles is rough. But, our results are consistent with the model of the QGMPs; the monopoles is strongly suppressed in the states arising from weakly coupled glasma (in the QGMPs with high temperatures), while they are dominant in the states arising from the strongly coupled glasma (in the QGMPs with low temperatures).

Here we mention that the classical gauge fields of the glasma are present when the gluon occupation number $\propto \alpha_s(Q_s)^{-1}$ in the color glass condensates is much larger than unity. That is, the coherence of the gluons is present for $\alpha_s(Q_s) \ll 1$. It is realized for large saturation momentum $Q_s \gg \Lambda_{QCD}$. In the range, the classical statistical field theories are applicable to the decay of the glasma, resulting in the gluon production. When Q_s becomes smaller, the coherence of the gluons gets worse. The strongly coupled glasma we discuss are characterized by large gauge couplings, but we expect that the coherence of the gluons still holds. We may assume that the classical gauge fields of the glasma are present even for large gauge couplings such as $\alpha_s \lesssim 1$ (or $Q_s \gtrsim \Lambda_{QCD}$); the occupation number is of the order of unity for $\alpha_s \simeq 1$. Therefore, the phenomenological model of the monopoles, which would be valid for large gauge coupling such as $\alpha_s \gtrsim 1/2$, can be applied to the decay of the classical gauge fields of the glasma.

III. APPLICABILITY OF SCHWINGER MECHANISM

Our production mechanism of gluons and monopoles is Schwinger mechanism, that is, they are generated as pair production[13, 14] under the background color electric and magnetic fields. We assume that the background gauge fields are spatially homogeneous and are pointed into the identical directions, both in real and color spaces. The gauge fields decrease with the pair production of the color charged particles. Furthermore, we assume that the field strength of color electric and magnetic fields are initially identical; $gE = gB = Q^2$. We have a parameter Q representing the strength of the gauge fields. We should point out that the energy densities of the gluons and monopoles produced are given by Q^4/g^2 , since the energies of the gauge fields are transformed into the energies of the particles.

As we show below, most of the gluons produced by Schwinger mechanism are the ones called as Nielsen-Olesen unstable modes[22]. The modes arise when classical color magnetic fields are present. Their presence implies instabilities[23] of the gauge fields and has been discussed in several numerical simulations[15–18] using inhomogeneous background gauge fields. The growth rates γ of the exponentially growing unstable modes $\sim \exp(\gamma t)$ found in the simulations correspond to Q in the present paper, i.e. $\gamma = Q$. That is, we describe the instabilities arising under inhomogeneous background gauge fields as instabilities arising under the homogeneous background gauge fields[23]. The field strengths of the homogeneous gauge fields are appropriately chosen so as to give rise to the identical growth rates to the ones obtained in the numerical simulations with the inhomogeneous background gauge fields. The description using such homogeneous gauge fields may be considered as a mean field approximation for gauge fields with general inhomogeneous configurations. In general, the parameter Q is much less than real saturation momenta Q_s of glasma. The calculation of the gluon production in the Schwinger mechanism is only reliable in the glasma with large Q such as $\alpha_s(Q) \ll 1$, since the gluons must weakly interact with each other for our approximation to be valid.

When we naively apply it to the glasma with small Q ($\alpha_s(Q) \sim 1$), we find that the electric fields decay so slowly that the gluon production hardly arise. But the application is not suitable to the glasma. Then we need to see how the electric fields decay after the rapid decay of the magnetic fields. We show in the section VII that the energies of the electric fields dissipate in the monopole plasma without the gluon production. Thus, the result of the gluon suppression is valid.

On the other hand, we assume an effective field theoretical model of monopoles in order to calculate their productions in the Schwinger mechanism. The model describes dual-superconductors[19, 20] in which quark confinement is realized with monopole condensations. We apply it to the analysis of the states in which gluons strongly couple with each other and the monopoles weakly couple with each other. The QGMPs with low temperatures as well as the prethermalized states produced by the decay of weakly coupled glasma with small Q would be such states. But the model is not applicable to the states with high temperatures $T \gg T_c$ or the weakly coupled glasma with large Q , since the magnetic charge $g_m = 4\pi/g$ is large so that the monopoles strongly couple with each other. On the other hand, it is general consensus that the monopoles do not play any roles and are absent in weakly coupled QGPs. Our result

is consistent with the QGMPs; the monopoles is strongly suppressed in the prethermalized states arising from the glasma with large Q , while they are dominant in the prethermalized states arising from the glasma with small Q .

When we naively apply the model to the glasma with large Q ($\alpha_s(Q) \ll 1$), we find that the magnetic fields decay so slowly that the monopole production hardly arise. But the application is not suitable to the glasma. As we show in the section VII, the energies of the magnetic fields dissipate in the gluon plasmas without the monopole production. Thus, the result of the monopole suppression is valid.

IV. MODELS OF GLUONS AND MONOPOLES

First we explain our model. We consider gluons in SU(2) gauge theory with the background color electric and magnetic fields given by $\vec{E}_a = \delta_{a,3}(0,0,E)$ and $\vec{B}_a = \delta_{a,3}(0,0,B)$; $a = 1, 2, 3$. They are supposed to be spatially homogeneous and collinear both in the real and color spaces. The gauge fields are represented by the diagonal component of the gauge potential $A_\mu \equiv A_\mu^{a=3}$. Under the background fields, the off-diagonal components $\Phi_\mu \equiv (A_\mu^1 + iA_\mu^2)/\sqrt{2}$ perpendicular to A_μ^3 behave as charged vector fields. When we represent SU(2) gauge potentials A_μ^a using the variables A_μ and Φ_μ , Lagrangian of SU(2) gauge fields is written[23] in the following,

$$L = -\frac{1}{4}F_{\mu,\nu}^2 - \frac{1}{2}|D_\mu\Phi_\nu - D_\nu\Phi_\mu|^2 - ig(\partial_\mu A_\nu - \partial_\nu A_\mu)\Phi^{\dagger\mu}\Phi^\nu + \frac{g^2}{4}(\Phi_\mu^\dagger\Phi_\nu - \Phi_\nu^\dagger\Phi_\mu)^2, \quad (1)$$

with $F_{\mu,\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D_\mu = \partial_\mu - igA_\mu$. The gauge field A_μ represents both the background gauge fields E and B . We find that the fields Φ_μ represent charged vector fields with the anomalous magnetic moment described by the term $-ig(\partial_\mu A_\nu - \partial_\nu A_\mu)\Phi^{\dagger\mu}\Phi^\nu$. They also have standard interactions with the gauge fields A_μ through the covariant derivative D_μ . Therefore, it is easy to see that when the background magnetic field $B = \partial_1 A_2 - \partial_2 A_1$ is present, the gluons represented by the fields Φ^μ occupy the Landau levels and interact with each other through the term $\frac{g^2}{4}(\Phi_\mu^\dagger\Phi_\nu - \Phi_\nu^\dagger\Phi_\mu)^2$. The energies E_n of the gluons with spin parallel to the background magnetic field are given by $E_n^2 = (2n+1)gB - 2gB + p_z^2 = (2n-1)gB + p_z^2$ with integer $n \geq 0$ where p_z denotes momentum component parallel to \vec{B} . The modes effectively have imaginary mass $i\sqrt{2gB}$, which arises from the term of the anomalous magnetic moment. Thus, we find that the modes with $E_{n=0} = \sqrt{p_z^2 - gB}$ are unstable when $p_z^2 - gB < 0$; the gauge fields exponentially grow such that $\Phi_{NO} \sim \exp(t\sqrt{gB - p_z^2})$. The modes are called as Nielsen-Olesen unstable modes[22, 23] and are produced spontaneously under the magnetic field B . We can see from the exponential growth of the gauge fields Φ_{NO} that the modes with smaller $|p_z|$ are produced more abundantly. The fact indicates that the soft modes of the gluons are dominantly produced in the early stage of the glasma decay.

When the electric field E is present, the production is accelerated owing to the Schwinger mechanism. As we show later, when the background gauge fields are strong, the gluons are dominant decay products of the gauge fields. It comes from the imaginary mass $i\sqrt{2gB}$ of the gluons.

On the other hand, the energies E_n of the gluons with spins anti-parallel to \vec{B} are given by $E_n^2 = (2n+1)gB + 2gB + p_z^2 = (2n+3)gB + p_z^2$. The modes are stable and effectively have mass $\sqrt{2gB}$ arising from the term of the anomalous magnetic moment. They are produced only when the electric field is present.

Since the production of the gluons and monopoles eventually makes the background gauge fields E and B vanish, the effective masses of the gluons vanish. Thus, the gluons become massless after their production.

Our model of the monopoles[5] describing dual superconducting states[8, 19, 20] is given by

$$L = |D_\nu\phi|^2 - \lambda(|\phi|^2 - v^2)^2 = |D_\nu\phi|^2 + \mu^2|\phi|^2 - \lambda|\phi|^4 - \lambda v^4 \quad (2)$$

with $\mu^2 \equiv 2\lambda v^2$ and $D_\nu = \partial_\nu - ig_m A_\nu^d$, where the field ϕ represents the monopole. We denote magnetic charge $g_m = 4\pi/g$ and dual gauge potential A_ν^d . We should note that the monopoles have imaginary mass $i\mu$ around the state $\phi = 0$. Thus, the monopole field exponentially grows such that $\phi \sim \exp(t\sqrt{\mu^2 - \vec{p}^2})$. It implies that the monopoles are spontaneously produced in the state $\phi = 0$ even without color magnetic fields $\vec{B} = -\partial_0 \vec{A}^d - \vec{\partial} A_0^d$. Similarly to the case of Nielsen-Olesen unstable modes, the monopoles with soft modes $\vec{p}^2 \ll \mu^2$ are dominantly produced and condense to make a confining vacuum; $\langle\phi\rangle = \mu/\sqrt{2\lambda}$.

The state $\phi = 0$ arises immediately just after the high energy heavy ion collisions. According to a model of color glass condensate, only longitudinal color electric and magnetic fields are generated after the collisions. It implies that there is no overall magnetic charges $\langle\int d\vec{S} \cdot \vec{B}\rangle = 0$. Thus we may suppose that the state $\phi = 0$ is initially realized in the glasma. The spontaneous production of the monopoles begins just after the collisions. When the magnetic field

is present, the production of the monopoles is accelerated owing to the Schwinger mechanism. Furthermore, when color electric field E is present, the monopoles occupy Landau levels specified by integer $n \geq 0$. Their energies E_n^m are given by $(E_n^m)^2 = (2n+1)g_mE + p_z^2 - \mu^2$. Thus, when the background electric field g_mE is smaller than μ^2 , the monopoles in the lowest Landau level ($n = 0$) are spontaneously produced. On the other hand the spontaneous production does not arise when the electric fields are strong enough such as $g_mE > \mu^2$.

We show below with the explicit use of the parameter $\mu = 0.5\text{GeV}$ that the large amount of the monopoles are dominantly produced by the weak gauge fields with $\sqrt{gE} = Q < 0.4\text{GeV} < \mu$, while the production of the monopoles is suppressed for the strong gauge fields with $\sqrt{gE} > \mu = 0.5\text{GeV}$ since the spontaneous production does not arise. We find that the values of the imaginary mass μ control the critical field strength $gB_c = gE_c = Q_c^2$ beyond which the monopole production is suppressed.

Furthermore, we show that each of the monopoles abundantly produced has small kinetic energies $< 10\text{MeV}$ for very weak gauge fields such as $\sqrt{gE} < 0.3\text{GeV}$. The weaker gauge fields induce the production of much more abundant monopoles with smaller kinetic energies. That fact leads to large collision cross sections $S = \pi l^2$ between monopoles since the distance l is roughly given by solving the equation such as the potential energies g_m^2/l equal to the small kinetic energies of the monopoles; $S \propto 1/(\text{kinetic energy})^2$. Thus, after the decay of the magnetic fields with small Q , the electric fields would rapidly decay in the monopole plasmas because magnetic resistances ($\propto S$) are large for small kinetic energies of the monopoles.

V. EVOLUTION OF NUMBER DENSITIES OF GLUONS AND MONOPOLES

We now proceed to show how the number densities of the gluons and monopoles evolve with time. We first note that the color charged particles are accelerated by color electric or magnetic fields. Thus, the energies of these gauge fields decrease. When the number density n_g (n_m) of the gluons (monopoles) is given, the energies of the charged particles increasing with their acceleration in a period dt are given such that

$$dt \times n_g \times gE = -d\left(\frac{E^2}{2}\right) \quad \text{and} \quad dt \times n_m \times g_mB = -d\left(\frac{B^2}{2}\right). \quad (3)$$

(We have neglected a term associated with polarization current[14], which comes from quantum production of the particles. The term has been shown much smaller than the terms in eq(3) associated with conduction currents[14].) The equations govern the evolution of the electric E and magnetic B fields as well as the number densities n_g and n_m . In order to solve the equations we need to know the number densities as the functions of the gauge fields; $n_g(gE, gB)$ and $n_m(gE, gB)$. The number densities of the charged particles produced by the Schwinger mechanism, have been obtained numerically in the references[14, 24], in which approximate formulas have also been given. We use the formulas given in the references.

Before giving the number densities of the gluons and monopoles under the background gauge fields, we notice that the number density of a charged scalar field with mass m and charge g produced by Schwinger mechanism under the electric field E has been given[14] by

$$n(t) = \int \frac{d^3p}{(2\pi)^3} \exp\left(-\frac{\pi(m^2 + p_x^2 + p_y^2)}{gE}\right) = \int \frac{gE dp_z}{(2\pi)^3} \exp\left(-\frac{\pi m^2}{gE}\right) \simeq \frac{(gE)^2 t}{(2\pi)^3} \exp\left(-\frac{\pi m^2}{gE}\right), \quad (4)$$

where we have taken into account the allowed range $gEt > p_z > 0$ of the momentum p_z of the produced particles after the electric field is switched on at $t = 0$. That is, the production rate of the particles is proportional to $\exp(-\pi(m^2 + p_x^2 + p_y^2)/gE)$. The formula has been given for the particles with their momentum p_z much larger than m . Furthermore, it is valid only for the electric field E constant with time t . Hereafter, we assume the formula even for E varying with time t , as long as the variation is smooth. As we show below, the gauge fields smoothly decay up to a certain time t_c , but decay rapidly after t_c . Thus, we use the formula until the rapid decay starts. We evaluate the number density n_g and n_m of the particles produced by the decay of the gauge fields at the time t_c .

When we impose a magnetic field B in addition to the electric field E , the number density of the particles with energies $E_n = (2n+1)gB + p_z^2 + m^2$ is given by

$$n(t) = \frac{gEt \times gB}{(2\pi)^2} \frac{\exp\left(-\frac{\pi(m^2 + gB)}{gE}\right)}{1 - \exp\left(-\frac{2\pi gB}{gE}\right)}, \quad (5)$$

where the factor $gB/(2\pi)$ comes from the degeneracy of a Landau level and the factor $(1 - \exp(-\frac{2\pi gB}{gE}))^{-1}$ comes from the summation, $\sum_{n=0}^{\infty} \exp(-2n\pi gB/gE)$, where n denotes the Landau levels. We should note that the term $m^2 + gB$ in eq(11) corresponds to the term $m^2 + p_x^2 + p_y^2$ in eq(4). That is, the transverse components $p_x^2 + p_y^2$ is replaced by $(2n+1)gB$ and the transverse integral $\int d^2p/(2\pi)^2$ is replaced with the summation $gB/(2\pi) \sum_{n=0}^{\infty}$. Then, after performing the summation, we obtain the factor $(1 - \exp(-\frac{2\pi gB}{gE}))^{-1}$. In this way we can easily obtain the formula eq(11) with $B \neq 0$ by replacing corresponding terms in eq(4) with relevant ones.

Using the formula, we can derive the number densities of the gluons and monopoles in terms of the background gauge fields \vec{E} and \vec{B} . Only the difference between the scalar particles and the gluons (monopoles) lies in their masses. The gluons with spin parallel and the monopoles have imaginary masses, i.e. $i\sqrt{2gB}$ and $i\mu$, respectively. (The gluons with spin anti-parallel have the effective mass $\sqrt{2gB}$.) Then, by replacing the mass in the formula eq(11) with the relevant ones, we obtain

$$n_g = \frac{gEt \times gB}{(2\pi)^2} \frac{\exp(\frac{\pi gB}{gE}) + \exp(\frac{-3\pi gB}{gE})}{1 - \exp(-\frac{2\pi gB}{gE})}, \quad \text{and} \quad n_m = \frac{g_mBt \times g_mE}{(2\pi)^2} \frac{\exp(\frac{\pi(\mu^2 - g_mB)}{g_mB})}{1 - \exp(-\frac{2\pi g_mB}{g_mB})}, \quad (6)$$

where the first term with $\exp(\frac{\pi gB}{gE})$ in n_g represents the contribution of the gluons with spin parallel (Nielsen-Olesen unstable modes), while the second term with $\exp(\frac{-3\pi gB}{gE})$ does the one of the gluons with spin anti-parallel. These formulas have been explicitly obtained[24] in canonical formalism, in which the gluons and monopoles with imaginary masses $i\eta$ become real particles when the squares of the energies $E^2 = p_z^2 + (i\eta)^2 > 0$ are positive with large $p_z = \int_{-\infty}^{\infty} dt' gE$ or $p_z = \int_{-\infty}^{\infty} dt' g_mB$. (Particles with imaginary masses im are virtual, but they become real when the square of their energies $E^2 = p^2 - m^2$ is positive with large momentum $p > m$. The real particles can be properly treated in canonical formalism of quantum field theories.)

It is easy to see from the formulas that the gluons are dominant decay products when the initial values Q^2 of gB and gE are larger than μ^2 , while the monopoles are dominant ones when the initial values of gB and gE are smaller than μ^2 . This is because the gluon production is accelerated by the decrease of gE according to the factor $\exp(\pi gB/gE)$ in n_g , while the monopole production is done by the decrease of gB according to the factor $\exp(\pi\mu^2/gB)$ in n_m . The gluon (monopole) production makes the electric field gE (magnetic field gB) decrease.

Using the number densities eq(6) and the equation (3), we find the evolution equations of gE and gB ,

$$\frac{d(gE)}{d\tau} = -\frac{\alpha_s gE gB}{2\pi} \frac{\exp(\frac{\pi gB}{gE}) + \exp(\frac{-3\pi gB}{gE})}{1 - \exp(-\frac{2\pi gB}{gE})} \quad \text{and} \quad \frac{d(gB)}{d\tau} = -\frac{gE gB}{2\pi\alpha_s} \frac{\exp(\frac{\pi(\mu^2 - g_mB)}{g_mB})}{1 - \exp(-\frac{2\pi g_mB}{g_mB})} \quad (7)$$

with $\tau \equiv t^2$. We solve the equations with the initial conditions $gE(\tau=0) = gB(\tau=0) = Q^2$. These are equations governing the production of the gluons and monopoles and the decay of the background gauge fields. They are very rough approximate formulas for corresponding equations derived in the classical statistical field theories.

Up to now, we derive the evolution equations of the gauge fields in SU(2) gauge theory. In the case of SU(3) gauge theory, we have three types of the off-diagonal gluons[25] and magnetic monopoles. The gluons are described by the gauge fields,

$$\Phi_1^\nu = \frac{A_1^\nu + iA_2^\nu}{\sqrt{2}}, \quad \Phi_2^\nu = \frac{A_4^\nu + iA_5^\nu}{\sqrt{2}}, \quad \Phi_3^\nu = \frac{A_6^\nu - iA_7^\nu}{\sqrt{2}}, \quad (8)$$

where the indices a of A_a^ν denote color degrees of freedom. The gluons couple with the background color electric E and magnetic fields B in maximal Abelian space,

$$E^i = \delta^{i,z} E (\cos(\theta)\lambda_3 + \sin(\theta)\lambda_8) \quad \text{and} \quad B^i = \delta^{i,z} B (\cos(\theta)\lambda_3 + \sin(\theta)\lambda_8). \quad (9)$$

where the angle θ describes the direction of the gauge fields in the maximal Abelian space spanned by the diagonal Gell-Mann matrices λ_3 and λ_8 . The angle θ takes a value in a range $-\pi/6 \leq \theta \leq \pi/6$ owing to the Weyl symmetry. We take an average over the angle to obtain final results by assuming the uniform distribution in θ . The coupling constants of the gluons Φ_i are given by

$$g_1 = g \cos \theta, \quad g_2 = \frac{g(\cos \theta + \sqrt{3} \sin \theta)}{2}, \quad \text{and} \quad g_3 = \frac{g(\cos \theta - \sqrt{3} \sin \theta)}{2} \quad (10)$$

respectively. The each gluon couples with the gauge fields E and B with its coupling constant.

Similarly, the three types of monopoles ϕ_i ($i = 1 \sim 3$) couple with dual gauge fields $A_a^{d,\nu}$ through covariant derivative, $D_\nu \phi_i = (\partial_\nu + ig_{m,i} A_\nu^d) \phi_i$ where the magnetic charges are given by $g_{m,1} = g_m \cos \theta$, $g_{m,2} = g_m(\cos \theta + \sqrt{3} \sin \theta)/2$ and $g_{m,3} = g_m(\cos \theta - \sqrt{3} \sin \theta)/2$.

Therefore, we add all the contributions of the three types of the gluons and the monopoles to obtain the number densities n_g and n_m in SU(3) gauge theory,

$$\begin{aligned} n_g &= \sum_{i=1 \sim 3} \frac{g_i E t \times g_i B}{(2\pi)^2} \frac{\exp(\frac{\pi g_i B}{gE}) + \exp(\frac{-3\pi g_i B}{gE})}{1 - \exp(\frac{-2\pi g_i B}{gE})} = \frac{3}{2} \frac{g E t \times g B}{(2\pi)^2} \frac{\exp(\frac{\pi g B}{gE}) + \exp(\frac{-3\pi g B}{gE})}{1 - \exp(\frac{-2\pi g B}{gE})} \\ n_m &= \sum_{i=1 \sim 3} \frac{g_{m,i} B t \times g_{m,i} E}{(2\pi)^2} \frac{\exp(\frac{\pi(\mu^2 - g_{m,i} E)}{g_{m,i} B})}{1 - \exp(\frac{-2\pi g_{m,i} E}{g_{m,i} B})} \end{aligned} \quad (11)$$

where we used the formulae $g_i B/g_i E = gB/gE$, $g_{m,i} E/g_{m,i} B = g_m E/g_m B$ and $\sum_{i=1 \sim 3} g_i^2 = 3g^2/2$.

Therefore, the evolution equations are given by

$$\frac{d(gE)}{d\tau} = -\frac{(6 \cos \theta - \cos 3\theta)}{4} \frac{\alpha_s g E g B}{2\pi} \frac{\exp(\frac{\pi g B}{gE}) + \exp(\frac{-3\pi g B}{gE})}{1 - \exp(\frac{-2\pi g B}{gE})} \quad (12)$$

$$\begin{aligned} \frac{d(gB)}{d\tau} &= -\frac{gE gB}{2\pi\alpha_s^2} \left(\cos^3 \theta \exp\left(\frac{\pi\mu^2 \alpha_s}{gB \cos \theta}\right) + 2^{-3} (\cos \theta + \sqrt{3} \sin \theta)^3 \exp\left(\frac{2\pi\mu^2 \alpha_s}{gB(\cos \theta + \sqrt{3} \sin \theta)}\right) \right. \\ &\quad \left. + 2^{-3} (\cos \theta - \sqrt{3} \sin \theta)^3 \exp\left(\frac{2\pi\mu^2 \alpha_s}{gB(\cos \theta - \sqrt{3} \sin \theta)}\right) \right) \frac{\exp(-\frac{\pi g E}{gB})}{1 - \exp(-\frac{2\pi g E}{gB})} \end{aligned} \quad (13)$$

with the initial conditions $gE(\tau=0) = gB(\tau=0) = Q^2$. After solving the equations with θ fixed, we calculate the number densities $n_g(\theta)$ and $n_m(\theta)$, and take the average $\bar{n} \equiv \int_{-\pi/6}^{\pi/6} \frac{d\theta}{\pi/3} n(\theta)$.

VI. RESULTS

We have solved the equations numerically with the use of the value $\mu = 0.5$ GeV and the running coupling constant $\alpha_s(Q) = g^2/4\pi = \alpha_c(1 + \frac{9\alpha_c}{4\pi} \log(Q^2/T_c^2))^{-1}$ with $\alpha_c = 0.9$ and $T_c = 0.16$ GeV used in the reference[4]. We take four values $\theta = 0, \pi/12, \pi/9$ and $\pi/6$ and take average of the densities n_g and n_m over the values θ .

We show the number density of gluons n_g in Fig.1. We can see that the number density per Q^3 of the gluons n_g/Q^3 is very small when small $Q < 0.35$ GeV, but it grows as Q increases and approaches an approximate constant when large $Q > 0.5$ GeV. We also show the number density per Q^3 of magnetic monopoles n_m/Q^3 in Fig.1. We find that the monopole production is suppressed when large $Q > 0.5$ GeV, while the production is enhanced when small $Q < 0.4$ GeV.

We show the fraction of the monopoles $n_m/(n_g + n_m)$ in Fig.2. We find that the gluons are dominant decay products of the strong gauge fields with $Q > 0.5$ GeV $\sim 3T_c$, while the monopoles are dominant ones of the weak gauge fields with $Q < 0.4$ GeV $\sim 2T_c$. The dominance of the gluons (monopoles) comes from the factor $\exp(\pi g B/gE)$ in n_g ($\exp(\pi\mu^2/gB)$ in n_m) when gE (gB) decreases with the production of the gluons (monopoles).

These features shown in Fig.1~Fig.3 are the very similar to those of the QGMPs recently proposed [4] if we identify Q as temperatures T , although the decay products do not interact with each other and are never thermalized in our discussions. There are no affirmative reasons for the identification $T \sim Q$. But we would like to point out that the energy density of the gluons and monopoles is given by Q^4/g^2 , while the energy density of thermalized massless particles is given by $\pi^2 f_{\text{eff}} T^4/30$; f_{eff} denotes the number of the species of the massless particles. Thus, it is not unreasonable to adopt the identification $Q \simeq gT(f_{\text{eff}}\pi^2/30)^{1/4}$.

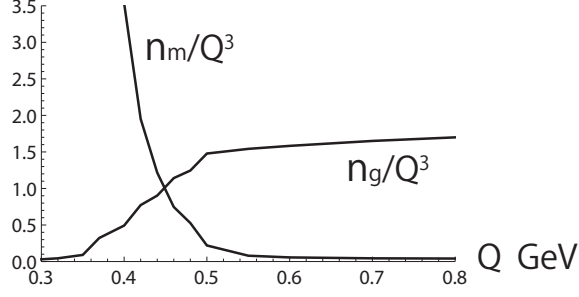


FIG. 1: The number density of gluons are suppressed when small $Q < 0.4\text{GeV}$, while the number density of monopoles is enhanced.

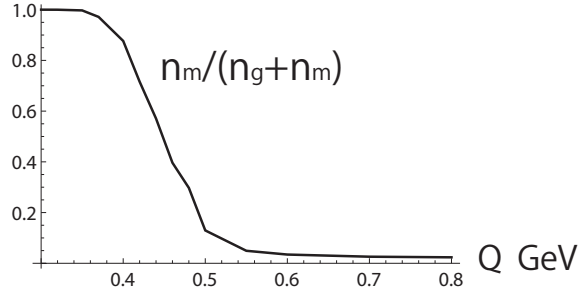


FIG. 2: The monopoles are dominant when small $Q < 0.4\text{GeV}$, while the gluons are dominant when large $Q > 0.5\text{GeV}$.

Using these results, we can see how the average kinetic energies $\epsilon_{g,m}$ of the gluons and the monopoles behave when Q changes. In order to see it, we note the energy conservation $n_g \times \epsilon_g + n_m \times \epsilon_m \simeq Q^4/g^2$. When Q is large, the gluons are dominant products and the average kinetic energy ϵ_g of the gluons behaves such that $\epsilon_g \sim Q$, since n_g/Q^3 is approximately constant for large Q . On the other hand, when Q is small, the monopoles are dominant products and the kinetic energy ϵ_m becomes smaller as Q becomes smaller. But we note that Q must be larger than Λ_{QCD} or T_c . Thus we can not take the limit $Q \rightarrow 0$. Thus, we wish to see how small the kinetic energies of the monopoles are. For the purpose, we use the formulas n_m in eq(11) even with small p_z , although the formulas are only valid for the large momentum $p_z > \sqrt{gB} = Q$ ($p_z > \mu$). We show in Fig.3 an average kinetic energy of a monopole given by $\epsilon_m \equiv |p_z| = \int_0^{t_c} dt n_m(t) g_m B(t) / n_m(t_c)$ where t_c denotes a time at which the densities n_m have been evaluated. The energy is the one acquired by a monopole as a result of the acceleration by the magnetic field B . We find that ϵ_m is approximately ten times smaller than Q for small Q . As we show below, the small kinetic energy of the monopoles causes large magnetic resistance of the monopole plasmas.

We show in Fig.4 and Fig.5 how E and B decay with time t . When large $Q > 0.6\text{GeV}$, the electric field fast decays while the magnetic field slowly decays. On the other hand, when small $Q < 0.4\text{GeV}$, the magnetic field fast decays while the electric field slowly decays. In particular, we should note that the gauge fields smoothly decay in the

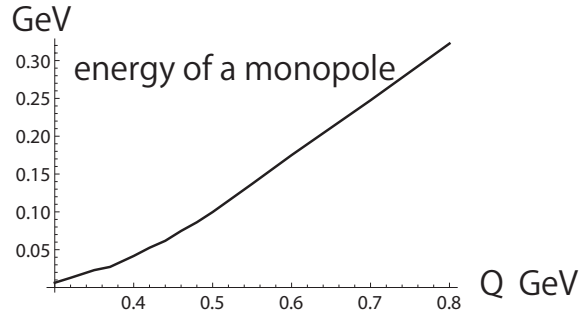


FIG. 3: The average kinetic energy of a monopole becomes smaller as Q becomes smaller. Using the energy conservation $n_m/Q^3 \times (\text{kinetic energy}) \simeq Q/g^2$, we can see how the kinetic energy decreases with Q .

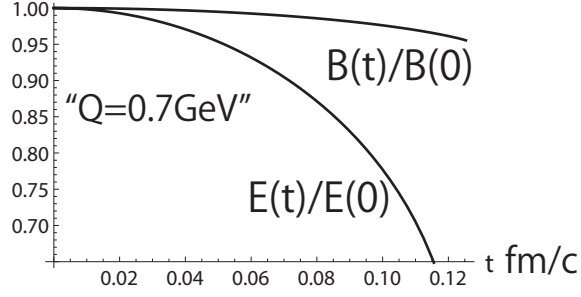


FIG. 4: At $Q = 0.7\text{GeV}$, the electric field fast decays, while the magnetic field slowly decays.

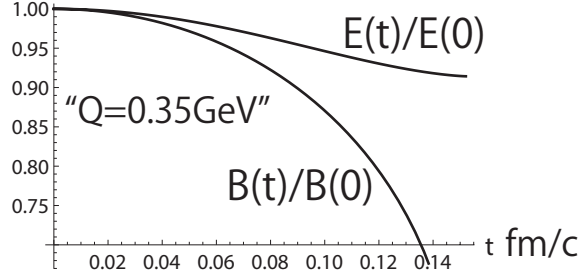


FIG. 5: At $Q = 0.35\text{GeV}$, the electric field slowly decays, while the magnetic field fast decays.

beginning and then they start to rapidly decay at a time t_c as shown in Fig.6 where we use the unit t^2 in horizontal axis to represent more clearly how rapid they decay. The time t_c has been used in the evaluation of $n_g(t = t_c)$ and $n_m(t = t_c)$.

Finally, we show in Fig.7 how the magnetic field B decays after the electric field E vanishes when $Q = 0.7\text{GeV}$. Obviously, the decay proceeds very slowly compared with the decay of the electric field shown in Fig.4. Similarly we can show that the magnetic fields rapidly decay at first and then the electric fields slowly decay when $Q = 0.35\text{GeV}$. As we have stated before, the slow decays of the remaining gauge fields are not correct because the models of the gauge fields are not applicable to their decays. Namely, the decay of the magnetic fields with large Q or small $\alpha_s(Q) \ll 1$ can not be discussed in the model of dual superconductors where the monopoles strongly interact with each other. Similarly, the decay of the electric fields with small Q or $\alpha_s(Q) \sim 1$ can not be discussed in the perturbative model of the gluons. We show in the next section that the remaining gauge fields rapidly vanish owing to the dissipation of their energies in the background gluon or monopole plasmas which are produced at first in the Schwinger mechanism. Thus, their decays do not change the result that the gluons are dominant decay products of the weakly coupled glasma and the monopoles are dominant ones in the strongly coupled glasma.

We have used the parameter $\mu = 0.5\text{GeV}$ in the calculation. Physical quantities such as n_g or n_m depend on μ . When we use different values μ , the whole behaviors of n_g/Q^3 and n_m/Q^3 in Q do not change. But the point $Q_c(\mu)$ at which $n_g(Q_c)/Q_c^3$ is equal to $n_m(Q_c)/Q_c^3$ is different. For example, when μ becomes larger than $\mu = 0.5\text{GeV}$, $Q_c(\mu)$ becomes larger than $Q_c(\mu = 0.5\text{GeV})$. Namely, the monopole dominance over the gluons arises at larger Q than

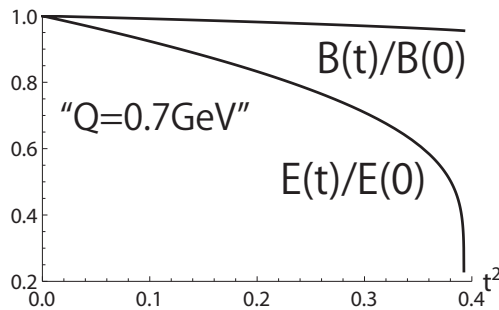


FIG. 6: At $Q = 0.7\text{GeV}$, we can see that the rapid decay of the electric field starts at around $t^2 \sim 0.38(\text{fm}/c)^2$.

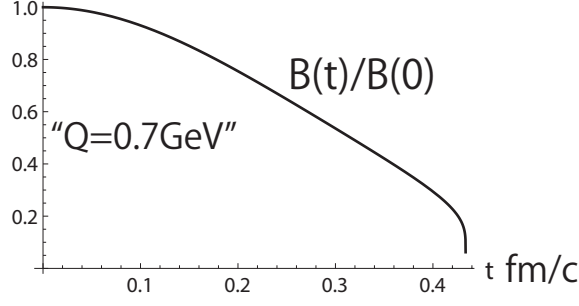


FIG. 7: We depict how the magnetic field slowly decays at $Q = 0.7\text{GeV}$ after the electric field vanishes $E(t \simeq 0.12\text{fm}/c) = 0$.

$Q_c \simeq 0.45\text{GeV}$ shown in Fig.2. The large imaginary mass of the monopoles enhances the spontaneous production of the monopoles owing to the factor $\exp(\pi\mu^2/gB)$ in n_m .

In our previous paper[12] we have discussed the decay of the gauge fields based on the classical statistical field theory, where we have used the values $\mu = 0.7\text{GeV}$ and $Q = 0.34\text{GeV}$. We have found that the magnetic fields vanish in a very short time $< 0.1\text{fm}/c$, while the electric fields decay very slowly. These results are consistent with the present analysis.

VII. DISCUSSION AND CONCLUSION

We have shown that when Q is large such as $Q > 0.5\text{GeV}$, the color electric fields rapidly decay into the gluons at first and then, the remaining color magnetic fields slowly decay into the monopoles. But, the decay mechanism of the magnetic fields is not reliable for strong magnetic fields with large Q (in other words, large g_m). Here, we would like to discuss how the remaining magnetic fields rapidly decay without the monopole production. As we show below, the gluon plasmas produced by the decay of the electric fields have color electrical conductivities σ proportional to Q for large Q . Then, the magnetic fields B vanish in the plasmas within a time of the order of Q^{-1} according to the Ampere's law $\partial_t B = \partial_x^2 B / \sigma$, since they typically possess the momenta Q in reality, $\partial_x^2 B \sim -Q^2 B$. (Although we have assumed the spatial homogeneity of B , it typically has momenta of the order of Q or Q_s .) Therefore, the magnetic fields rapidly decay for large Q without the monopole production after the electric fields decay. The fact does not change our result that the gluons are dominant decay products of the weakly coupled glasma with large Q .

We explain that the conductivities are of the order of Q for large Q . The conductivities are roughly given by $g^2 n_g t_f / \epsilon$ where ϵ denotes an average kinetic energy of a gluon and t_f does mean free time of gluons. t_f is defined by l/v with mean free path l and velocity $v = 1$ of gluons. On the other hand, the mean free path l is obtained in terms of the collision cross section $S = \pi r^2$ of the gluons such as $l = 1/(n_g S)$ where r is determined by equating the potential energy g^2/r with the kinetic energy ϵ_g . It is easy to see that S is of the order of Q^{-2} and $t_f = l \sim Q^{-1}$ since $\epsilon_g \sim Q$ and $n_g \sim Q^3$ for large Q as we have shown in Fig.1. (Here we note the energy conservation $n_g \times \epsilon_g \simeq Q^4/g^2$.) Therefore, we find that σ is of the order of Q for large Q .

On the other hand, we have shown that when Q is small but larger than Λ_{QCD} , the magnetic fields rapidly decay into the monopoles at first and then, the electric fields slowly decay into the gluons. But the decay mechanism of the electric fields is not reliable for the weak electric fields with small Q (in other words, large g). We would like to discuss that the electric fields rapidly vanish without the gluon production. They would decay owing to the large magnetic resistance of the monopole plasmas, in other words, small magnetic conductance σ_m . Namely, the monopoles produced by the decay of the magnetic fields have smaller kinetic energies as Q becomes smaller. Then the collision cross section $S = \pi r^2$ among the monopoles becomes larger as Q becomes smaller. That is, the cross section is determined by solving the equation such as the potential energy g_m^2/r equal to the kinetic energy ϵ_m . Thus, $S = \pi(g_m^2/\epsilon_m)^2$. This implies that the monopole plasmas have small magnetic conductance for small Q . Actually, the magnetic conductance σ_m is given by $g_m^2 n_m t_f / \epsilon_m \simeq g_m^2 / (\epsilon_m v S) \simeq \sqrt{2\epsilon_m M} / (2\pi g_m^2)$ where we assume $\epsilon_m = Mv^2/2$ with the mass M and velocity v of the monopoles. Although the mass of the monopoles is imaginary when they are produced, the monopoles would acquire real mass M after their production. Thus, the decay time $\tau \sim Q^{-2} \sigma_m$ of the electric fields is approximately estimated such that $\tau \simeq 0.04\text{fm}/c$ when $\epsilon_m = 0.05\text{GeV}$, $Q = 0.4\text{GeV}$, $M = 0.4\text{GeV}$ and $g_m = 1$. Although the mass M of the monopoles after their productions is unknown and the estimation is rough, our result indicates that the electric fields rapidly decay in the monopole plasmas. Therefore, the monopoles are dominant decay products of the weakly coupled glasma with small Q , since the decay of the electric fields does not produce the gluons.

We have shown that the dominant decay products are gluons when large $Q > 3T_c = 3 \times 0.16\text{GeV}$, while they are monopoles when small $Q \simeq (1 \sim 2)T_c$. These dominant decay products remain the main components after their thermalization as proposed in the model of QGMPs. The dominant decay products is determined by the comparison between the production rate $\propto \exp(gB/gE)$ in eq(6) of Nielsen-Olesen unstable modes and the rate $\propto \exp((\mu^2 - g_m E)/g_m B)$ in eq(6) of monopoles with imaginary mass. When the initial values $gE = gB = Q^2$ are larger than $(\mu^2 - g_m E)$, the Nielsen-Olesen unstable modes are dominantly produced in the initial stage of the production. Then, gE decreases faster than gB , which accelerates the production of the unstable modes. Thus, the dominant products are gluons when large Q . On the other hand, when the initial values $gE = gB = Q^2$ are smaller than $(\mu^2 - g_m E)$, the monopoles are dominantly produced. Then, gB decreases faster than gE , which accelerates the production of the monopoles. This is our production mechanism of the dominant particles.

Although we have not quantitatively discussed the momentum distribution of the prethermalized gluons and monopoles, we can qualitatively discuss the dominance of the soft gluons with $p^2 = p_z^2 + p_T^2 \ll Q^2$ produced in the decay of the glasma with large Q . This is because the Nielsen-Olesen modes with smaller longitudinal momentum p_z^2 , which grow as $\exp(t\sqrt{gB - p_z^2})$, are produced more abundantly and their typical transverse momentum $p_T^2 \sim gB$ vanish as gB vanishes with the dissipation in the gluon gas. We should remember that the fields of the modes are given such that $\Phi_{NO} \propto \exp(-x_t^2 gB/4)$ with the transverse coordinates x_t . Therefore, we find that the gluons are mainly composed of soft modes after the decay of the glasma. Similarly, the soft modes of the monopoles are dominantly produced in the decay of the glasma with small Q . (As a result, the monopole condensation may arise since the soft modes with almost zero momentum are mainly produced in the limit $Q \rightarrow T_c$ as numerically shown in eq(VI). This leads to the quark confinement.) The result of the soft gluon production is consistent with the previous studies[26] using classical statistical lattice simulations.

It has recently discussed[27] by using classical statistical simulations that the topological transition associated with the Chern-Simons number N_{cs} is enhanced in the early stage of the weakly coupled glasma evolution. That is, the number N_{cs} rapidly increases (or decreases) in the stage. Since dN_{cs}/dt is proportional to $\int d^3x \vec{E} \cdot \vec{B}$, it is easy to see in our analysis that the rapid decay of the electric fields leads to the rapid change of the number N_{cs} . Similarly, we can show that the rapid topological transition may arise in the early stage of the strongly coupled glasma evolution in which the magnetic fields rapidly decay. In this way we can understand the result of the elaborate numerical simulations[27] simply by using the Schwinger mechanism.

Using the model of the dual superconductors of the monopoles, we have shown that the gluons are dominant decay products of the weakly coupled glasma with large Q , while the monopoles are dominant ones of the strongly coupled glasma with small Q . Although our evolution equations of n_g and n_m is very rough, the roles of the monopoles in the strongly coupled QCD physics is clarified. Our results support the significance[4, 6] of the monopoles in the strongly coupled QGPs. More rigorous treatment of the monopoles in these strongly coupled QCD physics with low temperatures $T > T_c$ or small saturation momenta $Q_s > \Lambda_{QCD}$ is needed to confirm their roles mentioned above.

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